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DIOPHANTINE ANALYSIS.

85. Proposed by A. H. BELL, Hillsboro, Ill.

Given $x^2 - 85\frac{1}{2}y^2 = 5$. What is the value of x and y in whole numbers?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

$$x^2 - 85\frac{1}{2}y^2 = 5, \quad 4x^2 - 341y^2 = 20, \quad x^2 = 5 + \frac{341}{4}y^2.$$

$$\text{Let } y = 2z. \quad \therefore x^2 = 5 + 341z^2.$$

Let $z = 2$. Then $x = 37$, $y = 4$ are the least integral values.

86. Proposed by B. F. FINKEL, A.M., M.Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

Prove that $x^2 + 1457 \equiv 0 \pmod{2389}$ is insoluble.

Solution by J. W. YOUNG, Oliver Graduate Scholar in Mathematics, Cornell University, Ithaca, N. Y., and H. S. VANDIVER, Bala, Pa.

We are to prove that -1457 is a quadratic non-residue of the odd prime 2389 . Using Legendre's symbol, we must show that

$$\left(\frac{-1457}{2389}\right) = -1.$$

$$\text{We have } \left(\frac{-1457}{2389}\right) = \left(\frac{-1}{2389}\right) \left(\frac{31}{2389}\right) \left(\frac{47}{2389}\right).$$

$$\left(\frac{-1}{2389}\right) = +1, \text{ since } 2389 \text{ is of the form } 4n+1.$$

By the law of reciprocity, we have $\left(\frac{31}{2389}\right) = \left(\frac{2389}{31}\right) = \left(\frac{2}{31}\right) = +1$, since in the first place 2389 is of the form $4n+1$, and since, secondly, 31 is of the form $8n \pm 1$.

$$\left(\frac{47}{2389}\right) = \left(\frac{2389}{47}\right) = \left(\frac{39}{47}\right) = -\left(\frac{47}{39}\right) = -\left(\frac{8}{39}\right) = -\left(\frac{2^2}{39}\right) \left(\frac{2}{39}\right) = -\left(\frac{2}{39}\right) = -1,$$

since 47 and 39 are both of the form $4n-1$, and 31 is of the form $2n \pm 1$.

$$\text{Therefore, finally, } \left(\frac{-1457}{2389}\right) = (+1)(+1)(-1) = -1.$$

Also solved by G. B. M. ZERR.

AVERAGE AND PROBABILITY.

103. Proposed by LON C. WALKER, A. M., Assistant Professor of Mathematics, Leland Stanford Jr. University, Palo Alto, Cal.

A circle is drawn at random both in magnitude and position, but so as to lie wholly on the surface of a given semi-circle. Show that the chance that a radius drawn at random in the semi-circle will cut the circle is

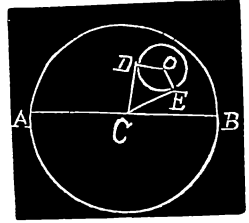
$$\frac{4}{3\pi - 4} \left(1 - \frac{1}{\pi} - \frac{2}{\pi} \log 2 \right).$$

Let C be the center of the semi-circle, O the center of the random circle, CD, CE tangents from C to random circle.

Also let $AC=r$, $CO=x$, $OD=z=OE$, $\angle ACO=\theta$, $\angle DCO=\varphi$, p =required chance.

The limits of θ are 0 and $\frac{1}{2}\pi$; of x , 0 and $r/(1+\sin\theta)$ $=x'$, and $r/(1+\sin\theta)=x'$ and r ; of z , 0 and $x\sin\theta$, and 0 $r-x$.

Then $p=2\varphi/\pi$, $\varphi=\sin^{-1}(z/x)$.



$$\begin{aligned} \therefore p &= \frac{\frac{2}{\pi} \int_0^{\frac{1}{2}\pi} \left[\int_{x'}^r \int_0^{r-x} \sin^{-1}(z/x) x dx dz + \int_0^{x'} \int_0^{x\sin\theta} \sin^{-1}(z/x) x dx dz \right] d\theta}{\int_0^{\frac{1}{2}\pi} \left[\int_{x'}^r \int_0^{r-x} x dx dz + \int_0^{x'} \int_0^{x\sin\theta} x dx dz \right] d\theta} \\ &= \frac{72}{\pi r^3 (3\pi - 4)} \int_0^{\frac{1}{2}\pi} \left[\int_{x'}^r \int_0^{r-x} \sin^{-1}(z/x) x dx dz + \int_0^{x'} \int_0^{x\sin\theta} \sin^{-1}(z/x) x dx dz \right] d\theta \\ &= \frac{72}{\pi r^3 (3\pi - 4)} \int_0^{\frac{1}{2}\pi} \left[\int_{x'}^r \left[(r-x) \sin^{-1}\left(\frac{r-x}{x}\right) + \sqrt{(2rx-r^2)-x} \right] x dx \right. \\ &\quad \left. + \int_0^{x'} (\theta \sin\theta + \cos\theta - 1) x^2 dx \right] d\theta \\ &= \frac{4}{\pi (3\pi - 4)} \int_0^{\frac{1}{2}\pi} \left[2 - \frac{3\theta}{(1+\sin\theta)^2} - \frac{(2+3\sin\theta+\sin^2\theta)\cos\theta}{(1+\sin\theta)^3} \right] d\theta \\ &= \frac{4}{\pi (3\pi - 4)} (\pi - \frac{1}{2} - 2\log 2) = \frac{1}{3\pi - 4} (4 - \frac{1}{\pi} - \frac{8}{\pi} \log 2). \end{aligned}$$

104. Proposed by LON C. WALKER, A. M., Assistant Professor of Mathematics, Leland Stanford Jr. University, Palo Alto, Cal.

In a given sphere two radii are drawn at random, and a point taken in each at random. (1) Find the chance that the distance between the two points does not exceed the radius of the sphere. (2) Find the distance between them.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let P be one of the points on radius OB , D the second point on radius OD . Let $OB=r$, $OP=x$, $OC=OD=y$. $OE=z$.

Then $PD=\sqrt{(x^2+y^2+2xz)}$.

The first point lies on the surface of a sphere radius x . An element of surface of this sphere is $2\pi x dx$. The second point lies on the surface of the sphere radius y . An element of surface of this sphere is $2\pi y dy$.

Let $(r^2-x^2-y^2)/2x=z'$, p =chance, and Δ =the average distance. Then

